DISCUSSION OF MR. VAN TUYL'S PAPER.

By W. S. Schlauch.

Mr. President: I have listened with a great deal of interest to the paper just read, because for the past year and more I have been investigating the mathematical calculations involved in business. Visits to such institutions as the National City Bank, The Columbia Knickerbocker Trust Co., The Seamen's Bank for Savings, Spencer Trask and Company, fire and life insurance companies, exporting houses and department stores have revealed some interesting facts.

In the first place, let me say that if we are to train the boys and girls in the commercial high schools to occupy only clerical or recording positions, the only mathematics we should give them is a drill in the four fundamentals, interest, and discount. This would enable the student to keep a minor position, but would not enable him to seize an opportunity to move up into the class of employes who have to do with organization and control of business, as distinguished from mere recording or clerical work.

In one respect, I think the attack of Mr. Van Tuyl on elementary algebra of the conventional type is justified; namely, the subject matter found in the problems offered as applications of the algebraic theory. However, commercial algebra, specifically pointing out the applications of algebra to business, can be made more interesting to the student, and more effective as an engine for calculation, than what now passes under the name of business arithmetic.

To illustrate by instances that I found in my visits, certain short cuts in calculation employed by book-keepers and accountants are more easily mastered by students who have had the algebraic generalization than by those who have not. For example: A student who understands the algebraic theory of addition of fractions, has no trouble in seeing that

$$\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$$

leads to the short cut rule for adding fractions whose numerators are 1, as

$$\frac{1}{7}+\frac{1}{11}=\frac{7+11}{77}=\frac{18}{77}.$$

The algebra lays the emphasis on the *process* and shows by the use of letters which represent *any* numbers that the process can be *generally* applied.

Again, aliquot part multiplying of which Mr. Van Tuyl makes so much in his text-book, giving 12 pages to its treatment, can be unified for a student who has had algebra and condensed into the formula:

To multiply any number A by n/d of 10^k, add K zeros and take n/d of the result.

Thus, to multiply 328 by $37\frac{1}{2}$, or by $\frac{3}{8}$ of 10^2, add 2 zeros and take $\frac{3}{8}$ of the result.

In allowing interest on daily balances, at the end of the month, the book-keepers in the large banks take advantage of the algebraic principle of common factor addition:

	Balance $\times$ Rate $\times\ \frac{1}{365}$.
$3ax$	$35000 \times .02 \times \frac{1}{365}$
$5ax$	$45000 \times .02 \times \frac{1}{365}$
$7ax$	$72000 \times .02 \times \frac{1}{365}$
$4ax$	$48000 \times .02 \times \frac{1}{365}$
Sum, $19ax$	....
	....
	Sum of Balances $\times .02 \times \frac{1}{365}$

The boy who has studied only arithmetic has trouble in seeing *why* it is not necessary to extend each item and take the sum of the separate days' interests.

Even the subject of interest is mastered more completely when generalized in algebra than in arithmetic. Each case in arithmetic is presented to the student as a *particular* and *separate* case. Algebra generalizes all into a formula and teaches the student to solve the formula to derive the converse cases.

Thus, (1) $i=prt$

and (2) $a=p+prt$

comprehend the whole subject of simple interest.

In fact, the men in the discount cage of the largest bank in America follow formula (1) in deriving a short rule for discounting notes when the discount rate is announced in the morning. For example, when I was in the bank one afternoon, it was 4½ per cent., and their rule was derived as follows:

$$r = \frac{9}{200}, \quad t = \frac{1}{360} \quad \therefore \quad i = p \times \frac{9}{200} \times \frac{1}{360} = p \times \frac{1}{8000}.$$

Therefore, *point off 3 places and divide by 8 for a day's interest.* In discounting a note for 24 days, they pointed off 3 places and multiplied by 3. A 30 day note for \$45,000 was discounted:

$$\begin{array}{r l} & 45 \\ & \underline{30} \\ 8) & \overline{1350} \\ & \overline{\$168.50} \text{ Discount,} \end{array}$$

although the clerk did not write as many figures as are used here to show the process.

The point I am making here is, that the generalization enables the discount clerk to derive his own "Bankers' Rule," and he does not find the interest or discount by the "Bankers' 60 day Rule" and then take off ¼, as the business arithmetics teach him he should. His method is algebraic and shorter.

Tables and efficiency devices I found to be in common use where repeated calculations are involved, and often the principle of calculation by which the table was made was algebraic.

The bond calculators used tables in determining the basis price of a bond, but they were of the opinion than in a High School of Commerce the method of calculating the basis price should be taught, and that a student should be taught *how to make such a table,* so that it would not be merely a mechanical process when he used it. This calculation involves only geometrical progression to derive the formula for the present value of an annuity, and logarithms to save labor in using the formula. Most business arithmetics publish tables of present values of annuities and bond tables, but the student uses them without understanding how they are derived.

For example, the basis price that can be paid for a bond bearing 5 per cent. interest, maturing in 14 years, to yield 4 per

cent. *can* be calculated by arithmetic when no table of answers is available, by using *14 long division processes.* It is calculated algebraically as follows:

The difference of rates will yield an annuity of \$10, the present value of which can be used as a premium in purchasing the bond.

$$A_0 = \frac{s}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$

gives the present value of an annuity s, at rate r, due in n years.

Substituting,

$$A_0 = \frac{10}{.04}\left[1 - \frac{1}{1.04^{14}}\right]$$

the premium that can be paid for the \$1,000 bond bearing 5 per cent., to yield 4 per cent. interest.

Graphs are best studied algebraically. Many corporations now demand graphs, cartograms, pictograms and other illustrations of a quantitative kind. Algebra is needed more and more in business.

From algebraic theory, I have worked out an alinement chart by means of which a paymaster can calculate wages due when different rates for time and overtime are allowed, by simply connecting the hours time with the hours overtime by a line, and reading the total wages due on the axis of wages.

I might go on indefinitely, but the point I make is that all students in commercial high schools should have enough drill in the first year in arithmetic to bring them to the level of maximum efficiency in handling the fundamental operations, and then should take commercial algebra for the generalized control it gives over the number system, and because investments, interest, insurance, exchange and other commercial calculations can be best mastered from the algebraic side.

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